INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING
Int. J. Numer. Meth. Engng 2004; **59**:1473–1500 (DOI: 10.1002/nme.922)

Finite calculus formulation for incompressible solids using linear triangles and tetrahedra

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SUMMARY

Many finite elements exhibit the so-called 'volumetric locking' in the analysis of incompressible or quasi-incompressible problems. In this paper, a new approach is taken to overcome this undesirable effect. The starting point is a new setting of the governing differential equations using a finite calculus (FIC) formulation. The basis of the FIC method is the satisfaction of the standard equations for balance of momentum (equilibrium of forces) and mass conservation in a domain of finite size and retaining higher order terms in the Taylor expansions used to express the different terms of the differential equations over the balance domain. The modified differential equations contain additional terms which introduce the necessary stability in the equations to overcome the volumetric locking problem. The FIC approach has been successfully used for deriving stabilized finite element and meshless methods for a wide range of advective–diffusive and fluid flow problems. The same ideas are applied in this paper to derive a stabilized formulation for static and dynamic finite element analysis of incompressible solids using linear triangles and tetrahedra. Examples of application of the new stabilized formulation to linear static problems as well as to the semi-implicit and explicit 2D and 3D non-linear transient dynamic analysis of an impact problem and a bulk forming process are presented. Copyright © 2004 John Wiley & Sons, Ltd.

KEY WORDS: finite calculus; volumetric locking; finite element method; linear triangles; linear tetrahedra; static analysis; dynamic analysis

1. INTRODUCTION

Many finite elements exhibit the so-called 'volumetric locking' in the analysis of incompressible or quasi-incompressible problems in fluid and solid mechanics. Situations of this type are usual

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Received 14 August 2002
Revised 13 March 2003
Accepted 28 May 2003