Possibilities of finite calculus in computational mechanics

Eugenio Oñate*†

International Center for Numerical Methods in Engineering (CIMNE), Universidad Politécnica de Cataluña, Edificio C1, Gran Capitán s/n, 08034, Barcelona, Spain

SUMMARY

The expression ‘finite calculus’ refers to the derivation of the governing differential equations in mechanics by invoking balance of fluxes, forces, etc. in a space–time domain of finite size. The governing equations resulting from this approach are different from those of infinitesimal calculus theory and they incorporate new terms which depend on the dimensions of the balance domain. The new governing equations allow the derivation of naturally stabilized numerical schemes using any discretization procedure. The paper discusses the possibilities of the finite calculus method for the finite element solution of convection–diffusion problems with sharp gradients, incompressible fluid flow and incompressible solid mechanics problems and strain localization situations. Copyright © 2004 John Wiley & Sons, Ltd.

KEY WORDS: finite calculus; computational mechanics; stabilized methods; finite element method; incompressible flow; incompressible solids; strain localization

1. INTRODUCTION

It is well known that standard numerical methods such as the central finite difference (FD) method, the Galerkin finite element (FE) method and the finite volume (FV) method, among others, lead to unstable numerical solutions when applied to problems involving different scales, multiple constraints and/or high gradients. Examples of these situations are typical in the solution of convection–diffusion problems, incompressible problems in fluid and solid mechanics and strain or strain rate localization problems in solids and compressible fluids using the standard Galerkin FE method or central scheme in FD and FV methods [1,2]. Similar instabilities are found in the application of meshless methods to those problems [3–5].

The sources of the numerical instabilities in FE, FD and FV methods, for instance, have been sought in the apparent inability of the Galerkin FE method and the analogous central difference scheme in FD and FV methods, to provide a numerical procedure able to capture the different scales appearing in the solution for all ranges of the physical parameters. Typical examples...